Titles and Abstracts

Participants

Hricha Acharya
Arizona State University

Carmen Arana
Université Paris Cité

Levi Axelrod
Binghamton University

Pavle Blagojević
FU Berlin & MI SANU

Mikhail Bludov
MIPT

Denys Bulavka
Dalhousie University

Wei-Chia Chen
Iowa State University

Minho Cho
KIAS

Ilkyoo Choi
Hankuk University & IBS

Suyoung Choi
Ajou University

Kai Fong Ernest Chong
SUTD

Michael Gene Dobbins
Binghamton University

Taehyun Eom
GIST

Florian Frick
Carnegie Mellon University

Penny Haxell
University of Waterloo

Andreas Holmsen
KAIST & IBS DIMAG

Bokhee Im
Chonnam National University

Jaewoo Jung
DGIST

YeongHun Jung
KAIST

Hyungu Kim
KAIST & IBS DIMAG

Hyobeen Kim
Chonnam National University

Jinha Kim
Chonnam National University

Minki Kim
GIST

Woojin Kim
KAIST

Minh Le
Ho Chi Minh City Open University

Dohyeon Lee
KAIST & IBS DIMAG

Participants

Seunghun Lee
Keimyung University

Martin Loebl
Charles University

Austin Longin
Iowa State University

Leonardo Martínez Sandoval
UNAM

Daniel McGinnis
Princeton University

Frédéric Meunier
École des ponts

Duc-Khanh Nguyen
VIASM

Ariadna Olvera Sampieri
UNAM

Eunku Park
DGIST

Jihye Park
Yeungnam University

Pavel Paták
Charles University

Heon Joon Pyun
CNU

Wei Rao
MIPT

Edgardo Roldán-Pensado
UNAM

Nikola Sadovek
MPI-CBG Dresden

Tseleung So
Pusan National University

Jongbaek Song
Pusan National University

Matěj Stehlík
Université Paris Cité

Martin Tancer
Charles University

Younghan Yoon
Ajou University

Jeonghyun Yoon
Chonnam National University

Semin Yoo
IBS DIMAG

Lu Yu
École des ponts

Shira Zerbib
Iowa State University

Bálint Zsigri
FU Berlin

Monday, June 29

9:30–10:00 *Registration*

10:00–11:00 **Suyoung Choi**
Real toric manifolds and their geometry

11:00–11:30 **Bálint Zsigri**
Intersection patterns via homotopy colimits

11:30–12:00 **Martin Tancer**
Non-acyclic spaces of line transversals

12:00–14:00 *Lunch*

14:00–15:00 **Denys Bulavka**
The Typical Algebraic Shifting of Graphs and Surfaces

15:00–15:30 **Younghan Yoon**
All full subcomplexes of Bier spheres

15:30–16:00 **Jaewoo Jung**
A Fröberg Type Theorem for Higher Secant Complexes

16:00–18:00 *Discussion*

Tuesday, June 30

9:30–10:00 *Coffee*

10:00–11:00 **Pavle Blagojević**
Towards the final Tverberg theorem

11:00–11:30 **Edgardo Roldán-Pensado**
Some old and new mass partition results

11:30–12:00 **Leonardo Martínez Sandoval**
A linear hypersimplicial Van Kampen-Flores theorem

12:00–14:00 *Lunch*

14:00–15:00 **Woojin Kim**
Combinatorial Aspects of Multi-parameter Persistent Homology

15:00–15:30 **Martin Loebl**
Connectivity Revisited

15:30–16:00 **Matěj Stehlík**
Vertex-critical subgraphs of Kneser graphs where topological bounds fail

16:00–18:00 *Discussion*

Wednesday, July 1

9:30–10:00 *Coffee*

10:00–11:00 **Shira Zerbib**
Chromatic number of 3-stable Kneser graphs

11:00–11:30 **Penny Haxell**
Algorithms for Independent Transversals and Reconfiguration

11:30–12:00 **Florian Frick**
Topology and sign-rank

12:00–14:00 *Lunch*

14:00–15:00 **Daniel McGinnis**
Multi-extremal Betti numbers and colored algebraic shifting

15:00–15:30 **Frédéric Meunier**
Fair and efficient allocation of indivisible items under category constraints

15:30–16:00 **Ariadna Olvera Sampieri**
Orthogonal star transversals in d-dimensional space

16:00–18:00 *Discussion*

Thursday, July 2

9:30–10:00 *Coffee*

10:00–11:00 **Nikola Sadovek**
On Geometric Transversal Theory and Intersection Patterns of Convex Sets

11:00–11:30 **Mikhail Bludov**
Colorful Carathéodory Theorem for Spanning k-trees

11:30–12:00 **Michael Dobbins**
MacPhersonians and pseudocircle arrangements

12:00–14:00 *Lunch*

14:00–15:00 **Kai Fong Ernest Chong**
Enumerative properties of Buchsbaum complexes

15:00–15:30 **Lu Yu**
Simplified proof of the equipartition result of Avvakumov and Karasev

15:30–16:00 **Carmen Arana**
Chromatic number of generalized Kneser graphs

16:00–18:00 *Discussion*

Friday, July 3

9:30–10:00 *Coffee*

10:00–11:00 **Duc-Khanh Nguyen**
Steenrod length and a problem of Vakil

11:00–11:30 **Minh Quang Le**
kNN: From a Topological Data Analysis (TDA) Perspective to a Topological Combinatorics Approach

11:30–12:00 **Minho Cho**
Colorful circuits and colorful topes in oriented matroids

12:00–14:00 *Lunch*

14:00–16:00 *Discussion*

16:00–18:00 *Dinner*

Chromatic number of generalized Kneser graphs

Carmen Arana

Université Paris Cité, Paris, France

Given a graph G, we define the generalized Kneser graph $KG(G, k)$ to be the graph whose vertices are the independent sets of size k in G, with edges between disjoint sets. As the notation suggests, this definition includes both the Kneser and Schrijver graphs as subcases, taking G to be the empty graph or the cycle respectively. Lovász [2] and Schrijver [3] showed respectively that in these two cases the chromatic number of $KG(G, k)$ is $n - 2k + 2$ where n is the order of G. We study which other graphs have this property and conjecture a characterization of such graphs. The case where $k = 2$ has already been proven by Daneshpajouh, Meunier, and Mizrahi [1].

Joint work with Penny Haxell, Tomáš Kaiser, and Matěj Stehlík.

References

[1] H. Daneshpajouh, F. Meunier, and G. Mizrahi, *Colorings of complements of line graphs*, Journal of Graph Theory **98**(2) (2021).

[2] L. Lovász, *Kneser's conjecture, chromatic number, and homotopy*, J. Combin. Theory Ser. A **25**(3) (1978), 319–324.

[3] A. Schrijver, *Vertex-critical subgraphs of Kneser graphs*, Nieuw Arch. Wisk. (3) **26**(3) (1978), 454–461.

Towards the final Tverberg theorem

Pavle V. M. Blagojević

Freie Universität Berlin, Berlin, Germany and Mathematical Institute SANU, Belgrade, Serbia

The theory of Tverberg-type results has a long, rich, and fascinating history, driven by the pursuit of fundamental problems and enriched by a wealth of beautiful ideas, theories, and techniques.

In this work, we develop a framework that points toward a complete classification of all strengthenings of the classical affine Tverberg theorem beyond the prime-power setting.

Colorful Carathéodory Theorem for Spanning k-trees

Mikhail Bludov

Moscow Institute of Physics and Technology, Moscow, Russia

The colorful Carathéodory theorem, proved by I. Bárány in 1982 [1], is a classical result in combinatorial geometry. It can be formulated as follows. Let Δ_V be a simplex on the vertex set V, let $V = V_1 \sqcup \cdots \sqcup V_{d+1}$ be a partition of vertices into $d+1$ nonempty classes, and let $K = V_1 * \cdots * V_{d+1}$ be the complex of colorful faces. If $f\colon \Delta_V \to \mathbb{R}^d$ is an affine map and $0 \in f(\Delta_{V_i})$ for all $i = 1, \ldots, d+1$, then there exists a face $\sigma \in K$ such that $0 \in f(\sigma)$.

Recently, P. V. M. Blagojević proposed a topological approach to the colorful Carathéodory theorem and proved a version with an additional constraint [2]. In particular, if $L \subset V_1 * V_2$ is a connected subcomplex containing all vertices and $K' = L * V_3 * \cdots * V_{d+1}$, then, under the same assumptions on f, there exists a face $\sigma \in K'$ such that $0 \in f(\sigma)$.

We propose a natural generalization of this result using spanning trees. Let X be a k-dimensional simplicial complex. We call a subcomplex $T \subset X$ a $\mathbb{Z}_2$-spanning k-tree if its $(k-1)$-skeleton coincides with the $(k-1)$-skeleton of X, if $H_k(T; \mathbb{Z}_2) = 0$, and if for every simplex $\sigma \in X \setminus T$ one has $H_k(T \cup \sigma; \mathbb{Z}_2) \neq 0$.

We prove the following theorem. Let $L \subset V_1 * \cdots * V_{k+1}$ be a spanning k-tree, where $1 \leq k \leq d$, and put $K' := L * V_{k+2} * \cdots * V_{d+1}$. Then for every affine map $f\colon \Delta_V \to \mathbb{R}^d$ such that $0 \in f(\Delta_{V_i})$ for all $i = 1, \ldots, d+1$, there exists a face $\sigma \in K'$ such that $0 \in f(\sigma)$.

This is joint work with Alexander Polyanskii. The authors thank R. N. Karasev for useful comments and discussions.

References

[1] I. Bárány, *A generalization of Carathéodory's theorem*, Discrete Mathematics **40** (1982), no. 2–3, 141–152.

[2] P. V. M. Blagojević, *Colourful Carathéodory's theorem plus a constraint*, arXiv:2509.01000v2, 2025.

The Typical Algebraic Shifting of Graphs and Surfaces

Denys Bulavka

Dalhousie University, Halifax, Canada

We initiate a statistical study of Kalai's exterior algebraic shifting, focusing on concentration phenomena for random triangulations of a fixed space. First, for a uniform n-vertex refinement of any given graph G, we show that asymptotically almost surely (a.a.s.) its exterior algebraic shifting is an explicit shifted graph depending only on n and the Betti numbers of G.

Next, for any given compact connected Riemannian surface S, sample n points independently at random according to the volume measure, and consider the a.a.s. unique Delaunay triangulation. We prove that a.a.s. its exterior algebraic shifting is an explicit shifted complex depending only on n and the Euler genus of S, and in particular is area-rigid. In both results the expected shifted complex is a *homology lex-segment* complex, a notion we define combinatorially and characterize numerically à la Björner–Kalai.

As a tool to prove the result on surfaces, we prove a universality result on edge contractions: for every fixed surface triangulation K, every dense enough point set in the surface yields a Delaunay triangulation that edge contracts to K.

This talk is based on joint works with Eran Nevo and Yuval Peled [1, 2].

References

[1] D. Bulavka, E. Nevo, and Y. Peled, *Volume rigidity and algebraic shifting*, J. Combin. Theory Ser. B **170** (2025), 189–202.

[2] D. Bulavka, E. Nevo, and Y. Peled, *The typical algebraic shifting of graphs and surfaces*, in *42nd International Symposium on Computational Geometry (SoCG 2026)*, Leibniz Int. Proc. Inform. **367**, pages 25:1–25:15, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2026.

Colorful circuits and colorful topes in oriented matroids

Minho Cho

Korea Institute for Advanced Study (KIAS), Seoul, Republic of Korea

We list some rainbow problems on oriented matroids and other related objects, such as words with bounded alternation numbers or point configurations. As a partial result, we obtain a "rainbow tope" result which implies a rainbow problem on words. The key lemma is a common generalization of Sperner's theorem and Meshulam's lemma [5], each of which guarantees the existence of rainbow simplices in vertex-colored simplicial complexes under different assumptions.

We follow the same approach to analyze structures of rainbow (co)circuits of oriented matroids. Using a similar proof technique, we provide an oriented matroid version of Bárány's colorful conic Carathéodory theorem [1].

This is joint work with Frédéric Meunier and Seunghun Lee [3].

References

[1] I. Bárány, *A generalization of Carathéodory's theorem*, Discrete Math. **40** (1982), 141–152.

[2] K. Bérczi, T. Király, Y. Yamaguchi, and Y. Yokoi, *Rainbow arborescence conjecture*, preprint, arXiv:2412.15457.

[3] M. Cho, S. Lee, and F. Meunier, *Colorful circuits and colorful topes in oriented matroids*, preprint, arXiv:2509.13718.

[4] A. F. Holmsen, *The intersection of a matroid and an oriented matroid*, Adv. Math. **290** (2016), 1–14.

[5] R. Meshulam, *The clique complex and hypergraph matching*, Combinatorica **21** (2001), no. 1, 89–94.

Real toric manifolds and their geometry

Suyoung Choi

Ajou University, Suwon, Korea

Toric topology studies spaces with torus actions by relating their topological and geometric properties to combinatorial data. Real toric manifolds, including small covers, form a real counterpart of toric manifolds and carry locally standard actions of elementary abelian 2-groups. Although their definition is parallel to that of complex toric manifolds, their geometry is often more subtle and less rigid.

In this talk, I will give an overview of real toric manifolds from the viewpoint of toric topology. I will explain how combinatorial structures control their topology, and discuss several geometric aspects such as orientability, asphericity, symplectic structures, and related classification problems. I will also describe recent developments and examples illustrating how real toric manifolds reveal phenomena that do not appear in the complex toric setting.

Enumerative properties of Buchsbaum complexes

Kai Fong Ernest Chong

Singapore University of Technology and Design (SUTD), Singapore

A d-dimensional simplicial complex Δ is defined to be *Buchsbaum* if it is pure, and if for every non-empty k-dimensional face F of Δ, the reduced homology of the link of F vanishes for all $i < d - k - 1$. In particular, Cohen–Macaulay complexes and homology manifolds are examples of Buchsbaum complexes.

In this talk, we present several enumerative results on the h-vectors and Betti numbers of Buchsbaum complexes. We generalize Klee's Dehn–Sommerville equations to the case of Buchsbaum complexes, stated in terms of a new weighted analog of h-vectors. Using a geometric interpretation, we also generalize Stanley's monotonicity theorem on the h-vectors of Cohen–Macaulay complexes to an analogous result on the h'-vectors of Buchsbaum complexes. These results are proven using a unified approach, via the study of spectral sequences corresponding to the rigidity double complex associated to a Buchsbaum complex.

MacPhersonians and pseudocircle arrangements

Michael Gene Dobbins

Binghamton University, Binghamton, New York, USA

MacPhersonians are simplicial complexes on oriented matroids, which are combinatorial analogues of real Grassmannians. A long-standing conjecture says that each MacPhersonian is homotopy equivalent to the corresponding Grassmannian. Pseudolinear Grassmannians are spaces of weighted topological representations of oriented matroids, and these are each homotopy equivalent to the corresponding Grassmannian in rank 3. I will present a good cover of the rank 3 pseudolinear Grassmannian with nerve complex isomorphic to the order complex of the corresponding MacPhersonian. This confirms the conjecture in rank 3.

Topology and sign-rank

Florian Frick

Carnegie Mellon University, Pittsburgh, USA

The sign-rank of a ± 1-matrix M is the smallest rank of a matrix A over the reals, where every entry of A has the same sign as the corresponding entry of M. Sign-rank is a fundamental concept in theoretical computer science, for example for communication complexity bounds. It is closely related to the VC dimension. Recently, new topological methods have been developed for concepts related to the VC dimension [1], including sign-rank [3]. I will present general topological bounds [2] that resolve the problem of determining the sign-rank of gap Hamming distance matrices.

This is joint work with K. Hosseini and A. Vasileuski.

References

[1] B. Chornomaz, S. Moran, and T. Waknine, *Spherical dimension*, Proceedings of Thirty Eighth Conference on Learning Theory, PMLR 291:1259–1313, 2025.

[2] F. Frick, K. Hosseini, and A. Vasileuski, *A $\mathbb{Z}_2$-topological framework for sign-rank lower bounds*, arXiv:2604.01510, 2026.

[3] H. Hatami, K. Hosseini, and X. Meng, *A Borsuk–Ulam lower bound for sign-rank and its applications*, In Proceedings of the 55th Annual ACM Symposium on Theory of Computing, pages 463–471, 2023.

Algorithms for Independent Transversals and Reconfiguration

Penny Haxell

University of Waterloo, Waterloo ON, Canada

An independent transversal (IT) of a graph G with a given vertex partition P is an independent set in G consisting of one vertex in each of the parts of P. This is a very general notion, and many problems in mathematics can be expressed by asking whether a particular graph G with a particular choice of P has an IT. Various criteria involving properties of G and P are known that will guarantee the existence of an IT in G.

More broadly, one may wish to consider the space of all ITs for a given G and P, and investigate how they are related. For example, under what conditions is this space connected, in the sense that one can transition from any IT to any other via a sequence of single-vertex modifications? It was shown by Buys, Kang and Ozeki [1] that with conditions only very slightly stronger than those ensuring the existence of a single IT, this connectivity property also holds.

Here we consider the algorithmic version of this question, and show that under certain similarly minimal conditions, such reconfiguration paths in the space of all ITs can be found efficiently.

This represents joint work with Sahab Hajebi.

References

[1] P. Buys, R. Kang, and K. Ozeki, *Reconfiguration of independent transversals*, Random Structures and Algorithms **67(1)** (2025), e70025.

A Fröberg Type Theorem for Higher Secant Complexes

Jaewoo Jung

BRL-AGSTA, DGIST, Daegu, Republic of Korea

In this talk, we generalize the celebrated Fröberg's theorem, which classifies the simplicial complexes of Leray number one, to embedded joins of copies of a simplicial complex which we call the (higher) secant complexes. We investigate combinatorial phenomena parallel to geometric ones observed for higher secant varieties of minimal degree. Our main result characterizes, by means of forbidden induced subgraphs, simplicial complexes Δ for a fixed integer $p \geq 1$ such that the Stanley-Reisner ring $S(\sigma_q \Delta)$ satisfies property $N_{q+1,p}$. This is a joint work with Dr. Junho Choe at KIAS.

References

[1] J. Choe and J. Jung, *A Fröberg Type Theorem for Higher Secant Complexes*, preprint, 2025.

Steenrod length and a problem of Vakil

Duc-Khanh Nguyen

Vietnam Institute for Advanced Study in Mathematics, Hanoi, Vietnam

We give an explicit combinatorial description of the function $f(n)$ governing the Steenrod length of real projective spaces $\mathbb{RP}^n$. This function arises in stable homotopy theory through the action of Steenrod squares on mod-2 cohomology and is closely related to the ghost length, which measures the minimal number of wedges of spheres required to construct a space up to homotopy. Building on the directed graphs T_n introduced by Vakil to encode degree constraints for Steenrod operations, we interpret $f(n)$ as the length of the longest directed path starting at n. Using this framework, we resolve a question posed by Vakil by deriving concrete combinatorial formulas for $f(n)$ in terms of binary classes and a distinguished family of integers, which we call Vakil numbers.

References

[1] D.-K. Nguyen, *Steenrod length and a problem of Vakil*, arXiv:2110.01672 (2026), 1–23.

[2] J. D. Christensen, *Ideals in triangulated categories: phantoms, ghosts and skeleta*, Advances in Mathematics **136**(2) (1998), 284–399.

[3] R. Vakil, *On the Steenrod length of real projective spaces: finding longest chains in certain directed graphs*, Discrete Mathematics **204**(1–3) (1999), 415–425.

Combinatorial Aspects of Multi-parameter Persistent Homology

Woojin Kim

KAIST, Daejeon, South Korea

This talk provides an introduction to persistent homology and its extension to multi-parameter persistence. In particular, we discuss how the classical combinatorial concept of Möbius inversion gives rise to the notion of the *generalized persistence diagram* for multi-parameter persistent homology, which naturally extends the celebrated persistence diagram for single-parameter persistent homology. If time permits, we will discuss the computational and descriptive complexity of the generalized persistence diagram, based on joint work with Donghan Kim and Wonjun Lee.

kNN: From a Topological Data Analysis (TDA) Perspective to a Topological Combinatorics Approach

Minh Quang Le

Ho Chi Minh City Open University, Ho Chi Minh City, Vietnam

Topological Data Analysis (TDA) is a rapidly growing field of applied mathematics in which techniques from computational and applied topology are used to extract structural information about the "shape" of data. TDA has been applied in numerous contexts, ranging from data science and machine learning to biology and neuroscience, visualization and dimensionality reduction [2, 1].

Topological Combinatorics is an area of mathematics that uses methods from topology to solve combinatorial problems. It studies how geometric and topological structures reveal information about discrete objects such as graphs, hypergraphs, simplicial complexes, and set systems [3].

In this work, the k-nearest neighbors (kNN) method is viewed as lying at the intersection of two complementary perspectives: topological data analysis (TDA) and combinatorics. From the standpoint of TDA, we investigate stability properties and establish convergence results. From the combinatorial perspective, we focus on the development of path homology.

Path homology has emerged through a sequence of developments, beginning with combinatorial constructions and subsequently extending to a general categorical framework. In this work, we consider a class of dynamical paths constructed from the nearest-neighbor structure associated with each vertex. Within this setting, we define the corresponding path complex and its associated homology theory. In particular, the kernel of the boundary operator is characterized in terms of distinguished (special) paths, while homology classes are represented by cyclic paths according to our formulation.

References

[1] M. Hajij, G. Zamzmi, and X. Cai, *Persistent homology and graphs representation learning*, arXiv:2102.12926, 2021.

[2] G. Carlsson, *Topology and data*, Bulletin of the American Mathematical Society (2009), 255–308.

[3] L. Caputi and G. Menara, *Eulerian magnitude homology: diagonality, injective words, and regular path homology*, arXiv:2503.06722, 2025.

Connectivity Revisited

Martin Loebl

Agate Centre, MFF UK, Charles University, Prague, Czech Republic

We initiate Repairable Optimisation, where we put together problems studied separately so far, broadly related to graph connectivity, e.g. flows and matchings viewed from an algorithmic game theory point of view. After a general overview I concentrate on one part of this new area which studies cooperative games where there exist groups of pre-aligned agents. For example, in a road network, the agents are cross-roads and pre-aligned groups are road-segments. For a set of datasets, the agents are attributes and each database which connects two (or more) attributes is pre-aligned. In this lecture, each pre-aligned group has size two.

Our goal is to find a way to determine the contribution of each individual local connection (pre-aligned couple) to the connectivity of the network, having an adversarial attack in mind or planning a defense of the network, or wanting to split the profits of the network between various owners of the individual local connections of the network. We model these contributions by Shapley values of the connectivity augmented 'local values' which are characteristic functions determined, e.g., by local incentives.

We link the concepts of potential and Shapley values of these connectivity augmented local values to the world of the Tutte polynomial and the partition function of the Potts model from statistical physics.

A linear hypersimplicial Van Kampen-Flores theorem

Leonardo Ignacio Martínez Sandoval

Faculty of Sciences, UNAM, Mexico City, Mexico

In this talk, we present a polyhedral generalization of the classic linear Van Kampen-Flores theorem. Historically, the traditional theorem establishes an upper bound on the neighborliness of projected simplices, proving that a $(2i+2)$-dimensional simplex cannot be linearly projected into $\mathbb{R}^{2i+1}$ while preserving its i-dimensional skeleton. In recent joint work with Padrol [1], we extend this framework to the (n,k)-hypersimplex $\Delta_{n,k}$ by completely characterizing the dimensions onto which it can be linearly projected while strictly preserving its i-skeleton. This hypersimplicial generalization reveals a far more complex structural landscape than the simplex case ($k = 1$), featuring three distinct geometric regimes based on the interplay between n, k, and i. This classification also settles an open question by Halman, Onn, and Rothblum regarding the convex dimension of complete uniform hypergraphs.

References

[1] L. Martínez-Sandoval and A. Padrol, *The convex dimension of hypergraphs and the hypersimplicial Van Kampen-Flores theorem*, Journal of Combinatorial Theory, Series B **149** (2021), 23–51.

Multi-extremal Betti numbers and colored algebraic shifting

Daniel McGinnis

Princeton University, Princeton, USA

Let $S = k[X_1 \cup \cdots \cup X_d]$ where the X_i are disjoint sets of variables and k is a field. Then S has a natural $\mathbb{Z}^d$-grading. If $M = \bigoplus M_{(a_1,\ldots,a_d)}$ is a $\mathbb{Z}^d$-graded finitely generated S-module, then $M_i = \bigoplus M_{(0,\ldots,a_i,\ldots,0)}$ is a $\mathbb{Z}$-graded S_i-module where $S_i = k[X_i]$. The multi-extremal Betti numbers of M are "built" from the extremal Betti numbers of the M_i; we will discuss the precise definition in the talk. Roughly speaking, the multi-extremal Betti numbers correspond to elements in the Koszul homology of M that can be written as the tensor product of elements in the Koszul homology of the M_i corresponding to their respective extremal Betti numbers.

We show that multi-extremal Betti numbers are preserved under taking the multi-graded generic initial ideal. As an application, we describe a relationship between the topological Betti numbers of a colored simplicial complex Δ and the topological Betti numbers of the simplicial complex obtained by applying colored algebraic shifting to Δ.

Fair and efficient allocation of indivisible items under category constraints

Frédéric Meunier

École nationale des ponts et chaussées, Champs-sur-Marne, France

We study the problem of fairly allocating indivisible goods and chores under category constraints. There are n agents and m items, partitioned into categories with capacity limits, and an allocation is feasible if every bundle respects these limits. For two agents, Shoshan et al. (2023) gave a polynomial-time algorithm for finding a Pareto-optimal allocation satisfying $\mathrm{EF}[1,1]$, a relaxed form of envy-freeness. Extending this beyond two agents was open.

We show that for any number n of agents, there exists a Pareto-optimal allocation such that each agent can be made envy-free by reallocating at most $\min\{k+1, n\}(n-1)$ items. We also give a polynomial-time algorithm for computing such an allocation when n is a constant. Our approach uses a new application of the Knaster–Kuratowski–Mazurkiewicz (KKM) lemma on a simplex of agent weights, which may be of independent interest. Joint work with Ayumi Igarashi.

Orthogonal star transversals in d-dimensional space

Ariadna Olvera Sampieri

Faculty of Sciences, UNAM, Mexico City, Mexico

The KKM theorem, due to Knaster, Kuratowski, and Mazurkiewicz [1], is a fundamental result in topology and fixed-point theory. This theorem and its many generalizations have found several applications in combinatorics and discrete geometry, giving rise to a variety of KKM-type results.

In this talk, we present an application of the colorful polytopal KKM theorem proved by Frick and Zerbib in [2] to geometric transversal theory. We prove a Helly-type result for finding a transversal *orthogonal star* to compact connected sets in $\mathbb{R}^d$, with Helly number $\kappa_d = O(\sqrt{d \ln d})$. We also discuss bounds on κ_d and some aspects of its optimality.

References

[1] B. Knaster, C. Kuratowski, and S. Mazurkiewicz, *Ein Beweis des Fixpunktsatzes für n-dimensionale Simplexe*, Fundamenta Mathematicae **14** (1929), 132–137.

[2] F. Frick and S. Zerbib, *Colorful coverings of polytopes and piercing numbers of colorful d-intervals*, Combinatorica **39** (2019), 627–637.

Some old and new mass partition results

Edgardo Roldán-Pensado

Centro de Ciencias Matemáticas, Universidad Nacional Autónoma de México, Morelia, Michoacán, Mexico

In this talk we consider some mass partition results related to the Grünbaum-Hadwiger-Ramos problem. When hyperplanes fail to simultaneously partition multiple measures we can utilize other, more flexible ways to partition space. Examples include the polynomial Ham-Sandwich theorem and the Yao-Yao partition theorem. We'll talk about these and other variants and present a new question related to the Yao-Yao partition theorem.

On Geometric Transversal Theory and Intersection Patterns of Convex Sets

Nikola Sadovek

Max Planck Institute of Molecular Cell Biology and Genetics, Dresden, Germany

In the first part of the talk, we present a recent result with Daniel McGinnis [1] in geometric transversal theory, establishing a necessary and sufficient condition for a family of convex sets in $\mathbb{R}^d$ to admit a k-dimensional affine transversal. We also describe a colorful (matroidal) extension obtained in the follow-up work [2].

These results connect naturally into the framework of Tverberg–Vrećica theory, where one seeks simultaneous guarantees on point-set partitions and the existence of k-dimensional transversals. In this context, we present complex analogues of central transversal theorems, Dol'nikov's transversal theorems, and Tverberg–Vrećica-type results [3].

In the final part of the talk, we discuss joint work in progress with Pavle Blagojević and Florian Frick [4] concerning the Tverberg admissible–prescribable conjecture, which is constrained to the case of point-transversals. The conjecture asks for a characterization of the tuples of face dimensions $(d_1, \dots, d_r)$ that are forced to appear in Tverberg r-partitions of sufficiently large point sets. Under the codimension-two assumption $d_i \leq d-2$ we resolve the remaining open cases ($\max_{i<j} |d_i - d_j| \geq 2$) of the topological version negatively, while verifying the affine counterpart. The construction of the counterexamples combines homotopy colimit techniques, which yield an equivariant map from the relevant configuration space to a sphere, with adaptations of the r-fold Whitney trick.

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Vertex-critical subgraphs of Kneser graphs where topological bounds fail

Matěj Stehlík

Université Paris Cité, Paris, France

Lovász [3] determined the chromatic number of the *Kneser graph* $\mathrm{KG}(n,k)$ by lower-bounding the chromatic number of any graph in terms of the connectivity of its neighbourhood complex. Since this seminal result, which relies crucially on the Borsuk–Ulam theorem, an entire menagerie of topological lower bounds on the chromatic number has emerged [4, 5].

Refining a subsequent proof by Bárány [1], Schrijver [6] identified the *Schrijver graph* $\mathrm{SG}(n,k)$ as a vertex-critical induced subgraph of $\mathrm{KG}(n,k)$ with the same chromatic number. Björner and de Longueville [2] later proved that Lovász's topological bound is tight for Schrijver graphs.

A natural question is whether Lovász's topological bound (or its modern variants) is tight for every vertex-critical induced subgraph of $\mathrm{KG}(n,k)$ with the same chromatic number.

In this talk, we answer the question in the negative by introducing a new family of vertex-critical induced subgraphs of $\mathrm{KG}(n,k)$ with the same chromatic number, for which the standard topological lower bounds are all strictly too low. Nonetheless, the Borsuk–Ulam theorem remains indispensable for establishing their chromatic number.

Joint work with Carmen Arana and Tomáš Kaiser.

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Non-acyclic spaces of line transversals

Martin Tancer

Department of Applied Mathematics, Charles University, Prague, Czech Republic

Cheong, Goaoc, and Holmsen conjectured that every connected component of the space of line transversals to a family of pairwise disjoint open convex sets in $\mathbb{R}^d$ is acyclic. We disprove this conjecture by showing that the homology may be nontrivial in any fixed dimension provided that d is large enough. More precisely, we show that for every $n \geq 1$ there is a finite family of pairwise disjoint open convex sets in $\mathbb{R}^{3n}$ such that the $(n-1)$st homology (over an arbitrary ring) of the space of line transversals to this family is nonzero. This is joint work with Haochi Jiang.

All full subcomplexes of Bier spheres

Younghan Yoon

Ajou University, Suwon, Republic of Korea

Bier spheres form an important family of simplicial spheres, including infinitely many non-polytopal examples. Recently, every Bier sphere was shown to admit a realization as a smooth complete fan, which highlights its relevance to toric geometry. In this talk, we study full subcomplexes of Bier spheres and determine their homotopy types explicitly. We also discuss applications to invariants associated with real toric varieties.

Simplified proof of the equipartition result of Avvakumov and Karasev

Lu Yu

École nationale des ponts et chaussées, Champs-sur-Marne, France

Envy-free cake cutting is a central problem in fair division theory and has been extensively studied over the past decades. A fundamental question is how to partition an interval into subintervals such that certain quantities associated with each piece are equally distributed.

A theorem of Avvakumov and Karasev states that for every continuous map assigning a real value to each subinterval of a closed interval vanishing on degenerate intervals, the interval admits a partition into any positive number of subintervals on which the function takes the same value. Their proof proceeds by induction and relies on a topological lemma. In this paper, we revisit the proof of this key lemma and give a more direct and combinatorial proof based on equivariant triangulations and Sperner-type labeling techniques. This leads to a shorter and more discrete proof of the equipartition theorem. Our approach can be useful for algorithmic and computational applications.

This is joint work with Frédéric Meunier.

Chromatic number of 3-stable Kneser graphs

Shira Zerbib

Iowa State University, Ames, Iowa, USA

For an integer $s \geq 2$, a subset $S \subseteq [n]$ is *s-stable* if $\min\{j-i,\, n+i-j\} \geq s$ for every $i, j \in S$ with $i < j$. Denote the set of all s-stable subsets of size k of $[n]$ by $\binom{[n]}{k}_{s\text{-stable}}$. Schrijver proved in 1978 that whenever $n \geq 2k$, the chromatic number of the Kneser graph $\mathrm{KG}\left(\binom{[n]}{k}_{2\text{-stable}}\right)$ equals $n - 2k + 2$. Generalizing this result, Meunier conjectured in 2011 that for all $n \geq sk$,

$$\chi\left(\mathrm{KG}\left(\binom{[n]}{k}_{s\text{-stable}}\right)\right) = n - sk + s.$$

The conjecture was proved by P. Chen for even s, and by J. Jonsson for $s \geq 4$ and large enough n. We prove the conjecture in the cases when $s = 3$ and n is large enough, or when $k = s = 3$. To this end, we prove a version of the Hilton–Milner theorem for s-stable sets. Joint with Wei-Chia Chen and Alex Parker.

Intersection patterns via homotopy colimits

Bálint Zsigri

Freie Universität Berlin, Berlin, Germany

Given a family of subsets of a fixed space, it is often of importance to describe the collection of subfamilies having a nonempty intersection. Both the Nerve Theorem and the well-known results about convex sets due to Helly, Berge, and Breen are concerned with such questions. Among others, the work of Meshulam [1], Montejano [2, 3], and Meunier & Montejano [4] shows that these theorems have homological analogues, where the assumptions and sometimes also the conclusions are replaced by the statements they would imply about homology groups.

We provide a systematic approach to these latter results, leading to further generalizations with weaker assumptions than previously. For this, we use the toolkit of homotopy colimits, which we extend by homologically flavored lemmas, relating homotopy colimits to order- and nerve complexes. This is joint work with Imre Bárány and Pavle Blagojević.

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Supported by
GIST, IBS DIMAG, Chonnam National University, and National Research Foundation of Korea
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